In the last few weeks, we have gone through the concepts of relative speed. You might be surprised to know that we can use the same concept to solve some clock problems too. The reason some clock problem can be tricky is that the hour hand and the minute hand move simultaneously so handling them separately is not easy. In such questions you can easily use relative speed i.e. speed of the minute hand relative to the hour hand. Let's try to understand this with the help of an example.

Example: In a circular clock, the minute hand is the radius of the circle. At what time is the smaller angle between the minute hand and the hour hand of the clock not divisible by 10?

I. 7:20

II. 4:30

III. 9:00

(A) Only I
(B) Only II
(C) I + II
(D) II+III
(E) I+II+III

Solution: First let me explain why the question is a little complicated. We need to find the smaller angles between the two hands of the clock at given times. We know that at 9'o clock the hour hand is at 9 and the minute hand is at 12. So the angle between the hour hand and the minute hand is 90 degrees. But what about 7:20? We know that at 7:20, the minute hand is at 4 but where is the hour hand? Is it exactly at 7? No, it is somewhere between 7 and 8. At 4:30, is the hour hand at 4 or mid way between 4 and 5? So you see there is a complication. You have to account for the little bit of distance covered by the hour hand too to get the angle between the hour hand and the minute hand.

Let's see the relevance of relative speed here: Minute hand covers 360 degrees in an hour i.e. it makes one full rotation. Or we can say that it covers 30 degrees in every 5 mins. On the other hand, the hour hand completes one rotation of 360 degrees in 12 hrs. Or we can say that it covers 360/12 = 30 degrees in an hour.
Speed of minute hand relative to hour hand is 360 – 30 = 330 degrees per hour (since they move in the same direction so the relative speed is the difference between their speeds).

Let's now try to find the angle between the hour hand and the minute hand at the given times. We will start with the easiest one.

III. 9:00

At 9 o clock, the hour hand is at 9 and the minute hand is at 12. The angle between the two hands is 90 degrees. 90 is divisible by 10. This was simple enough.

II. 4:30

At 4 o clock, the minute hand is at 12 and the hour hand is at 4 i.e. the minute hand is 120 degrees behind the hour hand. In half an hour, it covers 330/2 = 165 degrees. This means it covers the 120 degrees between them and further creates a gap of 45 degrees i.e. the minute hand is 45 degrees ahead of the hour hand now. The smaller angle between them is 45 degrees. 45 is not divisible by 10.

I. 7:20

At 7’o clock, the minute hand is at 12 and the hour hand is at 7 i.e. the minute hand is 210 degrees behind the hour hand (going clockwise). In 20 minutes (i.e. at 7:20), it makes up 330*20/60 = 110 degrees. Now the minute hand will be 210 – 110 = 100 degrees behind the hour hand. The smaller angle between them now is 100 degrees. 100 is divisible by 10.

So the smaller angle between the hands is not divisible by 10 at 4:30.

Answer (B)

I hope you see that the question has become quite simple. This is just one of the many applications of relative speed. You can obviously do it without using relative speed concepts too. The method you choose to use during the exam should be the one you are most comfortable with.